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Two-point correlators in the β-deformed $\mathcal{N} = 4$ SYM at the next-to-leading order

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Abstract: We compute two-point functions of lowest weight operators at the next-to-leading order in the couplings for the β-deformed $\mathcal{N} = 4\,\mathrm{SU}(N)$ SYM. In particular we focus on the CPO $\mathrm{Tr}(\Phi_1^2)$ and the operator $\mathrm{Tr}(\Phi_1\Phi_2)$ not presently listed as BPS. We find that for both operators no anomalous dimension is generated at this order, then confirming the results recently obtained at lowest order in [5]. However, in both cases a finite correction to the two-point function appears.

Keywords: AdS-CFT and dS-CFT Correspondence, Renormalization Regularization and Renormalons, Superspaces.

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Exactly marginal deformations of $\mathcal{N}=4$ SYM theory preserving $\mathcal{N}=1$ supersymmetry [1] have recently attracted much attention, in particular in connection with AdS/CFT correspondence [2]. In fact, the supergravity duals of the so-called β-deformations of $\mathcal{N}=4\,\mathrm{SU}(N)$ SYM theory have been found by Lunin and Maldacena in [3]. This opens the possibility of making a comparison between correlation functions of (un)protected operators in the two weak/strong coupling regimes of the theory, in a way similar to what has been done in the last few years for the undeformed $\mathcal{N}=4$ theory.

The spectrum of chiral primary operators (CPO) for the deformed theory has been found in [4]. They are classified according to the charge assignement with respect to three U(1) groups such that Φ_i is charged under the i-th group [3, 5]. For generic deformation parameters the chiral ring is given by operators $\mathrm{Tr}(\Phi_i^J)$, $i=1,2,3$, and $\mathrm{Tr}(\Phi_1^J\Phi_2^J\Phi_3^J)$.

In the $\mathcal{N}=4$ case the operators $\mathrm{Tr}(\Phi_i^J)$ and $\mathrm{Tr}(\Phi_i^{J-1}\Phi_k)$, $k\neq i$ are both CPO and related by a SU(3) R-symmetry transformation. Therefore they share the same properties: They do not acquire anomalous dimension and their 2- and 3-point functions are not corrected at the quantum level.

In the deformed case the two sets of operators belong to two different classes and they might undergo a different destiny. It is therefore compelling to investigate their quantum properties. This program has been undertaken in a very recent paper [5] where the authors have computed 2- and 3-point functions of some operators in the deformed superconformal field theory at the leading order in perturbation theory (see also [6]). In the case of CPO's they find that at this order the corresponding correlators have no radiative corrections. This is more than the expected property of having no anomalous dimension, and it seems to indicate that these operators are very similar to the CPO's of the undeformed $\mathcal{N}=4$ theory.
However, the most unexpected result of [5] is that the operator $\mathrm{Tr}(\Phi_1\Phi_2)$ has protected 2pt function at the lowest order, although it was not recognized as a CPO in the previous literature. The aim of our paper is to test these unexpected properties at the next-to-leading order in perturbation theory, in order to understand if they are an accident of the one-loop calculation or they signal an actual protection of this operator.

We concentrate on the lowest weight CPO $\mathrm{Tr}(\Phi_1^2)$ and $\mathrm{Tr}(\Phi_1\Phi_2)$ and compute their two-point functions at the next-to-leading order. We find that the unexpected nonrenormalization of $\mathrm{Tr}(\Phi_1\Phi_2)$ persists at the next-to-leading order. This result supports the idea that this operator should be included in the CPO classification [4, 3].[1]

However, we find that the 2pt functions for both operators get a finite correction, in contradistinction to the CPO's of the undeformed $\mathcal{N}=4$ case.

The most convenient setup to perform higher order calculations is the $\mathcal{N}=1$ superspace (we will use notations and conventions in [7]). In this framework the β-deformed theory is described by the following action

$$S[J,\bar{J}] = \int d^8z\, \mathrm{Tr}\left(e^{-gV}\bar{\Phi}_i e^{gV}\Phi^i\right) + \frac{1}{2g^2}\int d^6z\, \mathrm{Tr}\, W^\alpha W_\alpha +$$

[1] We thank Juan Maldacena for pointing out the possibility for this operator to be lacking in the present classification of CPO's for β-deformed theories.

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$$+ih\int d^6z\,\mathrm{Tr}(e^{i\pi\beta}\Phi_1\Phi_2\Phi_3 - e^{-i\pi\beta}\Phi_1\Phi_3\Phi_2) +$$
$$+ih^*\int d^6\bar{z}\,\mathrm{Tr}(e^{i\pi\beta}\bar{\Phi}_1\bar{\Phi}_2\bar{\Phi}_3 - e^{-i\pi\beta}\bar{\Phi}_1\bar{\Phi}_3\bar{\Phi}_2) + \int d^6z J\mathcal{O} + \int d^6\bar{z}\bar{J}\bar{\mathcal{O}} \qquad (1)$$

where h and β are complex couplings. The superfield strength $W_\alpha = i\bar{D}^2(e^{-gV}D_\alpha e^{gV})$ is given in terms of a real prepotential V and $\Phi_{1,2,3}$ contain the six scalars of the original $\mathcal{N}=4$ SYM theory organized into the $\mathbf{3}\times\bar{\mathbf{3}}$ of SU(3) $\subset$ SU(4). We write $V = V^aT_a$, $\Phi_i = \Phi_i^aT_a$ where T_a are SU(N) matrices in the fundamental representation. We have added to the classical action source terms for composite chiral operators generically denoted by $\mathcal{O}$ (J ($\bar{J}$) are (anti)chiral sources).

We perform the calculation of two-point correlators by following closely the procedure described in [8–10]. Here we briefly recall the main steps of the general prescription, while referring to those papers for a detailed discussion.

In euclidean space we introduce the generating functional

$$W[J,\bar{J}] = \int \mathcal{D}\Phi\mathcal{D}\bar{\Phi}\mathcal{D}V e^{S[J,\bar{J}]} \qquad (2)$$

for the n-point functions of the operator $\mathcal{O}$

$$\langle\mathcal{O}(z_1)\cdots\bar{\mathcal{O}}(z_n)\rangle = \left.\frac{\delta^n W}{\delta J(z_1)\cdots\delta\bar{J}(z_n)}\right|_{J=\bar{J}=0} \qquad (3)$$

where $z \equiv (x,\theta,\bar{\theta})$. The perturbative evaluation of the n-point function is equivalent to computing the contributions to $W[J,\bar{J}]$ at order n in the sources. Since we are interested in the two-point super-correlator for chiral operators of weight 2 we look for quadratic contributions of the form

$$W[J,\bar{J}] \to \int d^4x_1 d^4x_2 d^4\theta J\left(x_1,\theta,\bar{\theta}\right)\frac{F(g^2,|h|^2,\beta,N)}{[(x_1-x_2)^2]^{2+\gamma}}\bar{J}\left(x_2,\theta,\bar{\theta}\right). \qquad (4)$$

The x-dependence of the result is fixed by the conformal invariance of the theory at this order, whereas $F(g^2,|h|^2,\beta,N)$ signals possible finite quantum corrections and $\gamma = \gamma(g^2,|h|^2,\beta,N)$ is the anomalous dimension which the operators can acquire at the quantum level.

We work in momentum space, using dimensional regularization and minimal subtraction scheme. In n dimensions, with $n = 4-2\epsilon$, the naive dimension of the operators is continued to the value $2(1-\epsilon)$. The Fourier transform of the power factor in (4) is given by

$$\frac{1}{(x^2)^{2(1-\epsilon)+\gamma}} = 2^{2-2\gamma+2\epsilon}\pi^{2-\epsilon}\frac{\Gamma(-\gamma+\epsilon)}{\Gamma(2(1-\epsilon)+\gamma)}\int\frac{d^np}{(2\pi)^n}\frac{e^{-ipx}}{(p^2)^{-\gamma+\epsilon}}. \qquad (5)$$

By performing analytic continuation of $\Gamma(-\gamma+\epsilon)$ and expanding in powers of γ we obtain (see [10] for details)

$$\frac{1}{(x^2)^{2(1-\epsilon)+\gamma}} \sim C\left[\frac{1}{\epsilon}+\frac{\gamma}{\epsilon^2}+\frac{1}{\epsilon}O\left(\frac{\gamma^2}{\epsilon^2}\right)\right]\times\int\frac{d^np}{(2\pi)^n}\frac{e^{-ipx}}{(p^2)^{-\gamma+\epsilon}} \qquad (6)$$

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where C is a constant. In momentum space and dimensional regularization, the $\frac{1}{\epsilon}$ divergent term corresponds to the short distance singularity of the correlation function (4) for $x_1 \sim x_2$, whereas contributions $\frac{1}{\epsilon^2}$ signal the presence of an anomalous dimension. Therefore, in performing perturbative calculations in momentum space we will look for all the contributions to (4) that behave like $\frac{1}{\epsilon}$ and $\frac{1}{\epsilon^2}$, disregarding finite contributions (in x space they would give contact terms). Once the divergent terms are determined at a given order in the couplings, by anti-Fourier transforming back to the configuration space we can reconstruct an x-space structure as in (4) with a nonvanishing contribution to $F(g^2, |h|^2, \beta, N)$ and γ.

The basic rules of our strategy can then be summarized as follows: we consider all the two-point diagrams from $W[J, \bar{J}]$ with J and $\bar{J}$ on the external legs. We write the corresponding analytic expression by using Feynman rules as coming from the action (1) (after gauge-fixing, we work in Feynman gauge where $\langle VV \rangle = \frac{1}{\Box}$). Moreover we find convenient to rewrite the chiral superpotential as

$$-h\left(f_{abc}\cos\pi\beta + d_{abc}\sin\pi\beta\right)\int d^6z\,\Phi_1^a\Phi_2^b\Phi_3^c + \text{h.c.} \tag{7}$$

where $f_{abc} = -i\,\mathrm{Tr}(T_a[T_b, T_c])$ and $d_{abc} = \mathrm{Tr}(T_a\{T_b, T_c\})$.[2] Then we evaluate all combinatorics factors of a given diagram and compute the colour structure (we use conventions and identities listed in appendix A of [8]). Then we perform the superspace D-algebra following standard techniques reducing the result to a multi-loop momentum integral. Finally we compute its $\frac{1}{\epsilon}$ and $\frac{1}{\epsilon^2}$ divergent contributions (we refer to appendix B of [8] for the integrals).

Before entering the explicit calculation of correlators we need investigate the renormalization properties of the theory described by the action (1). In particular we are interested in the evaluation of the beta functions up to two loops in order to impose the condition of superconformal invariance at the quantum level (vanishing beta functions). For the $\mathcal{N} = 4$ SYM theory and in a superspace setup, two-loop beta functions were computed in [11] where one and two-loop diagrams contributing to the propagators and vertices can be found. The β-deformed theory differs from $\mathcal{N} = 4$ SYM only for the structure of the chiral vertex, while the propagators and the vector-chiral vertices are the same. Therefore, all the perturbative contributions from diagrams which do not contain chiral vertices are the same as for $\mathcal{N} = 4$ and we can read the results in [11]. What we are left with is the calculation of diagrams with chiral vertices.

At one-loop order the corrections to the propagators of the fundamental chiral fields are given in figure 1.

Computing the first diagram with the chiral vertex (7) and using known results for the second diagram we find the following divergent contribution to $\int d^8z\,\mathrm{Tr}(\Phi_i\bar{\Phi}_i)$

$$\frac{1}{(4\pi)^2\epsilon}2N\left[|h|^2\left(|\cos\pi\beta|^2 + \frac{N^2-4}{N^2}|\sin\pi\beta|^2\right) - g^2\right] \tag{8}$$

[2] We have slightly changed conventions with respect to our old papers [8 – 10] by rescaling the constants d_{abc} by a factor 2.

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Figure 1: One-loop contributions to the chiral propagator.

in agreement with [5]. This gives rise to an anomalous dimension for the chiral fields proportional to the square bracket in (8). According to the non-renormalization theorem for the $N = 1$ chiral superpotential [11, 12] the beta function for the chiral coupling is proportional to the anomalous dimension of the chiral fields. Therefore, one-loop superconformal invariance requires

$$\left[|h|^2\left(|\cos\pi\beta|^2 + \frac{N^2-4}{N^2}|\sin\pi\beta|^2\right) - g^2\right] = 0\,. \tag{9}$$

The same condition insures that also the gauge coupling beta function vanishes at this order. This follows from general renormalization properties of these theories [13], or it can be proven easily by a direct calculation [11] of the $V\Phi\bar{\Phi}$ 3pt function which, under the condition (9), turns out to be finite and identical to the one in the $\mathcal{N} = 4$ theory

$$\frac{g^3}{4}N\mathrm{if}^{\mathrm{abc}}\bar{\Phi}^i_a(q,\theta,\bar{\theta})\Phi^i_b(-p,\theta,\bar{\theta})\left(4D^\alpha\bar{D}^2D_\alpha + (p+q)^{\alpha\dot{\alpha}}[D_\alpha,\bar{D}_{\dot{\alpha}}]\right)V_c(p-q,\theta,\bar{\theta})\times$$
$$\times\int\frac{d^nk}{k^2(k-p)^2(k-q)^2}\,. \tag{10}$$

Moving to two loop order, first we compute the self-energy corrections to the chiral propagators. Exploiting the results for the $\mathcal{N} = 4$ case and recomputing the contributions of diagrams containing chiral vertices we find that, under the condition (9), the result is finite and coincides with the $\mathcal{N} = 4$ result [11]

$$-2g^4N^2\bar{\Phi}^i_a(p,\theta,\bar{\theta})\Phi^i_a(-p,\theta,\bar{\theta})p^2\int\frac{d^nq\;d^nk}{k^2q^2(k-q)^2(k-p)^2(p-q)^2} =$$
$$= -2g^4N^2\bar{\Phi}^i_a\left(p,\theta,\bar{\theta}\right)\Phi^i_a(-p,\theta,\bar{\theta})\frac{1}{(p^2)^{2\epsilon}}[6\zeta(3)+\mathcal{O}(\epsilon)]\,. \tag{11}$$

Therefore the condition (9) is sufficient to guarantee the vanishing of the anomalous dimension of the fundamental fields at this order. Again, according to general renormalization arguments [13] this should also imply the vanishing of beta functions, i.e. superconformal invariance at two loops.

Now armed with the condition (9) we compute the two-point function for the chiral primary operator $\mathrm{Tr}(\Phi_1^2)$ up to the next-to-leading (two-loop) order. This correlator has been recently computed at one-loop in [5] by using a component description of the theory in the WZ gauge. The result indicates that at lowest order non only the operator is not renormalized as it should be, but also there is no finite correction to the correlator. In a superspace language this result is very easy to reproduce. In fact at this order the only diagrams contributing are the ones in figure 2, where in the first diagram the one-loop

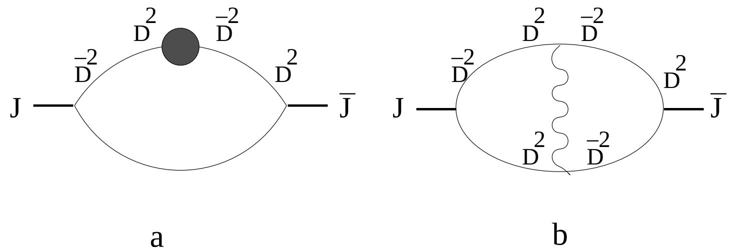

Figure 2: One-loop contributions to $\langle \mathrm{Tr}(\Phi_1)^2\, \mathrm{Tr}(\bar{\Phi}_1)^2 \rangle$.

correction to the propagator has been inserted. Since under the condition (9) this insertion vanishes the first diagram is trivially zero in the superconformal case. Once D-algebra has been performed the second diagram corresponds to a finite (not interesting for us) momentum integral. Thus we can conclude that at the lowest order the 2pt function does not get either finite corrections or anomalous dimension contributions.

At two loops the potentially divergent diagrams are given in figure 3 (we neglect diagrams which were shown to be finite in [8]).

In diagrams 3a and 3b the two-loop chiral self-energy (11) and the gauge-chiral vertex correction (10) have been inserted, respectively. The results for these diagrams can be read from the $\mathcal{N} = 4$ case, as well as the one for diagram 3c (see [8]). Extracting an overall factor

$$16 \frac{1}{(4\pi)^6} N^2(N^2-1) \int d^4p\ d^4\theta J(-p,\theta,\bar{\theta}) \bar{J}(p,\theta,\bar{\theta}) \tag{12}$$

we have

$$\begin{aligned} \text{figure } 3a &\rightarrow g^4 \zeta(3) \frac{1}{\epsilon} \\ \text{figure } 3b &\rightarrow -2g^4 \zeta(3) \frac{1}{\epsilon} \\ \text{figure } 3c &\rightarrow \frac{1}{2} g^4 \zeta(3) \frac{1}{\epsilon} \end{aligned} \tag{13}$$

Diagram 3d needs to be computed since it contains the new vertex (7). What changes is the color factor, while the D-algebra and the momentum integral are identical to the $\mathcal{N} = 4$ case. Again, extracting the overall factor (12) and using (9) we obtain (from now on we concentrate on the case of real β)

$$\begin{aligned} &\frac{1}{2}|h|^4 \left[\cos^4 \pi\beta - 6\frac{N^2-4}{N^2} \cos^2 \pi\beta \sin^2 \pi\beta + \frac{(N^2-4)(N^2-12)}{N^4} \sin^4 \pi\beta \right] \zeta(3) \frac{1}{\epsilon} = \\ &\qquad = \frac{1}{2}\left[g^4 + 8|h|^4 \frac{N^2-4}{N^2} \sin^2 \pi\beta \left(\frac{N^2-1}{N^2} \sin^2 \pi\beta - 1 \right) \right]\ \zeta(3) \frac{1}{\epsilon} \end{aligned} \tag{14}$$

where the condition (9) has been used once again. Summing up all the contributions we immediately see that the $\frac{1}{\epsilon}$ poles proportional to g^4 cancel in agreement with the $\mathcal{N} = 4$ case, whereas a new non vanishing contribution proportional to $|h|^4$ appears. This term is proportional to $\sin^2 \pi\beta$ and it vanishes in the undeformed limit ($\beta \rightarrow 0$, $|h|^2 \rightarrow g^2$). When

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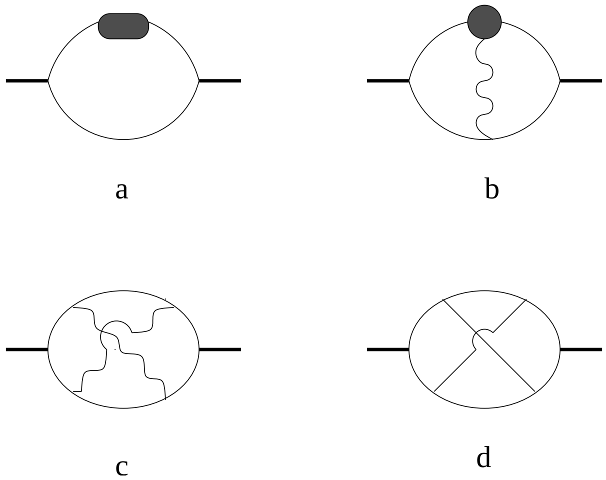

Figure 3: Two-loop contributions to $\langle \mathrm{Tr}(\Phi_1)^2\, \mathrm{Tr}(\bar{\Phi}_1)^2 \rangle$.

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we transform the result back to configuration space we end up with a nontrivial correction to the 2pt function of order $|h|^4$

$$\langle \mathrm{Tr}(\Phi_1^2)(z_1)\, \mathrm{Tr}(\bar{\Phi}_1^2)(z_2) \rangle_{2-\mathrm{loops}} \sim \frac{\delta^{(4)}(\theta_1-\theta_2)}{[(x_1-x_2)^2]^2} |h|^4 \frac{N^2-4}{N^2} \sin^2 \pi\beta \left(\frac{N^2-1}{N^2} \sin^2 \pi\beta - 1 \right). \tag{15}$$

We note that this correction survives even in the large-N limit.

Now we consider the operator $\mathrm{Tr}(\Phi_1\Phi_2)$ which in principle might renormalize. In [5] it has been shown that at the lowest order in perturbation theory this operator does not acquire anomalous dimension and its 2pt function is not corrected. This result can be easily reproduced in our language: The one-loop diagrams contributing to the two-point function for this operator are still given in figure 2. Since the first diagram is zero in the superconformal case while the second diagram is always finite, we do not get either corrections to the correlators or anomalous dimension contributions.

In order to investigate whether the absence of anomalous dimension is an accident of the one-loop calculation we compute the next-to-leading contributions to the 2pt function. The diagrams contributing to this correlator are still the ones given in figure 3 where now we have Φ_1 and Φ_2 fields coming out from the source vertices, plus the extra diagram in figure 4 where we have indicated explicitly the two possible contractions of Φ_1, Φ_2 and $\bar{\Phi}_1, \bar{\Phi}_2$.

In computing the contributions from figure 3 one easily realizes that what changes with respect to the previous calculation is the color factor of diagram $(3d)$, while D-algebra and momentum integrals remain the same. Therefore, as it immediately appears from eqs. (13), (14) we will still get at most $1/\epsilon$ divergences, so no contributions to the anomalous dimension arise from the diagrams in figure 3. The only potential source of $1/\epsilon^2$ contributions is the diagram in figure 4 since, after completion of the D-algebra, it gives rise to momentum loop integrals with self-energy subdivergences. However, there is a complete cancellation between the two contributions drawn there due to a sign change

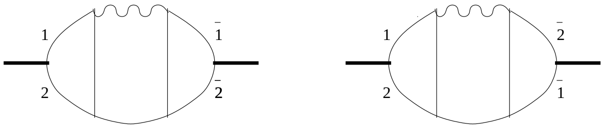

Figure 4: Additional two-loop diagram for $\langle \mathrm{Tr}(\Phi_1\Phi_2)\,\mathrm{Tr}(\bar\Phi_1\bar\Phi_2)\rangle$.

in the color structure produced by the exchange of $\bar\Phi_1$ with $\bar\Phi_2$. We then conclude that the diagram in figure 4 does not contribute and even at this order the operator $\mathrm{Tr}(\Phi_1\Phi_2)$ does not acquire an anomalous dimension. Our result confirms the lowest order [5] protection of this operator, then giving further support to its BPS nature.

Contributions $1/\epsilon$ from diagrams in figures $3a$, $3b$, $3c$ are still given in eqs. (13). From diagram $3d$, taking into account the two possible flavor configurations of the internal lines, we obtain

$$\frac{1}{2}|h|^4\left[\cos^4\pi\beta + 2\frac{N^2-4}{N^2}\cos^2\pi\beta\sin^2\pi\beta + \frac{(N^2-4)(N^2-12)}{N^4}\sin^4\pi\beta\right]\zeta(3)\frac{1}{\epsilon} = \\ = \frac{1}{2}\left[g^4 - 8|h|^4\frac{N^2-4}{N^4}\sin^4\pi\beta\right]\zeta(3)\frac{1}{\epsilon} \qquad (16)$$

where we have used (9). Again, we see that the g^4 contributions cancel, consistently with the $\mathcal{N}=4$ case. We are left with a contribution proportional to $|h|^4$ which gives a finite correction to the two-point function of the operator

$$\langle \mathrm{Tr}(\Phi_1\Phi_2)(z_1)\,\mathrm{Tr}(\bar\Phi_1\bar\Phi_2)(z_2)\rangle_{2-\mathrm{loops}} \sim \frac{\delta^{(4)}(\theta_1-\theta_2)}{[(x_1-x_2)^2]^2}|h|^4\frac{N^2-4}{N^4}\sin^4\pi\beta\,. \qquad (17)$$

This contribution vanishes in the undeformed limit and it is subleading in the large-N limit.

The results presented in this letter are part of a systematic analysis of correlators in the deformed $\mathcal{N}=4$ SYM theory [14].

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